

Nonlinear Frequency Shift in Acoustoelectric Domains in GaAs

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A Bömmel-Dransfeld technique is used to analyse the frequency spectrum in GaAs acoustoelectric domains up to 8 GHz. Parametric frequency conversion is found to be only dominant at the very onset of domain formation. In the strongly nonlinear regime, the amplified noise is mainly concentrated around the shifting frequency of maximum nonlinear gain. The shift of the maximum gain frequency is generally discussed by a theory of nonlinear ultrasound amplification. An evaluation of the measured frequency shifts by this theory leads to the simple relation $|x| \approx 2y$ between the domain electric field and the bunching rate.

1. Introduction

From Brillouin scattering studies¹⁻³ and current noise measurements⁴ it is known that the frequency distribution of highly amplified vibrational noise in acoustoelectric domains has maxima different from the frequency ω_m of maximum linear gain^{5,6}. These peaks have been explained by parametric amplification of subharmonic acoustic waves^{7,8}. On the other hand an indication of a continuous shift of maximum gain frequency has also been observed in the nonlinear regime^{3,4}.

Using a Bömmel-Dransfeld technique⁹ a new frequency analysis of the vibrations involved in an acoustoelectric domain up to 8 GHz is measured for GaAs. The different development with time of parametric processes and nonlinear effects in the amplification process could be resolved. In particular, the shift of the frequency of maximum gain has been detected and has been evaluated by a nonlinear theory¹⁰. The results leads to new ideas on the development of the domain frequency spectrum.

2. Experimental Technique and Results

A rod $1 \times 1 \times 10.8$ mm was cut along the [110] crystal direction from a 1.5Ω cm oxygen-doped GaAs material purchased from Monsanto Chemicals. The end

faces were polished optically flat and parallel. In-Te contacts were alloyed to the side faces close to the end faces in order to obtain approximately a homogeneous voltage drop down the rod. A high voltage pulse was applied to obtain travelling acoustic domains in the rod. A BÖMMEL-DRANSFELD method⁹ was used to detect the acoustic waves reflected at the flat end face of the specimen. The frequency spectrum, therefore, is measured at the time when the domain hits the end face of the specimen. In order to obtain different growth stages for the domain at this definite point of the specimen the applied voltage was varied between 700 and 750 V. Besides this the same sampling technique to analyse the frequency spectrum of the pulsed signals was used as described in the previous paper¹¹.

A typical result for a frequency spectrum is shown in Fig. 1. The nonlinear deviation from the ohmic current $\Delta I = 1.1$ A is used as a measure for the state of the nonlinearity. In intervals of 2 GHz different attenuators have been applied to avoid nonlinear effects in the detection unit of the frequency analyser. The power scale, therefore, must be multiplied by this attenuation factors if one compares the magnitude of the peaks in different frequency ranges. The shifted frequency $\tilde{\nu}_m$ of maximum acoustic power is observed at a frequency 2.5 GHz. All the other observed peaks are not related to this frequency but to the frequency of maximum linear gain at 3.6 GHz which could not be detected, being

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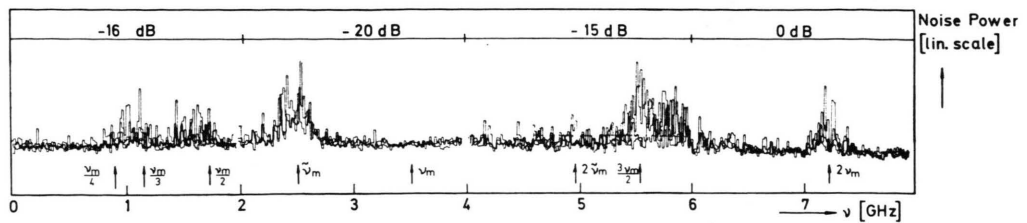


Fig. 1. Acoustic noise spectrum in the frequency range 0 to 8 GHz taken during domain formation at nonlinear current deviation $\Delta J = 1.1$ A.

in the same interval 2–4 GHz as the strong shifted peak at 2.5 GHz. As marked in the spectrum, peaks are observed at the subharmonic frequencies $\nu_m/2$, $\nu_m/3$, and $\nu_m/4$ to the frequency of maximum linear gain. Note that there also is a strong peak observed at $\frac{3}{2} \nu_m$ which can only be explained by mode-mixing of the fundamental frequency ν_m with the subharmonic $\nu_m/2$. The second harmonic is also observed at 7.2 GHz. No frequency shift could be detected for all these peaks. Only the small peak at 5 GHz follows the shift of the main peak and can thus be explained as second harmonic of the main frequency peak.

3. Discussion of the Experimental Results

The following interpretation for this behaviour of the frequency spectrum is suggested. At very early stages of domain formation the frequency of maximum linear net gain is negligibly shifted but pumps energy into subharmonic acoustic waves which again mix with the fundamental frequency to form the observed peaks at rational multiples of the linear fundamental frequencies. At a further developed stage of the domain the power of acoustic waves at frequency of maximum gain reaches a level at which the concentration nonlinearity causes three different effects:

1. The well-known shift of the frequency of maximum gain.
2. The saturation of harmonic generation.
3. A reduction of subharmonic generation¹¹.

At the more developed stage of the domain the noise peak at the frequency of maximum gain thus loses the coupling to the pumped subharmonics which from this time on propagate freely to the end surface where they are detected. This decoupling might be a consequence of the break-down of the Manley-Rowe relation which is inappropriate for a

highly active medium*. Because of 2. only a small peak is observed at the harmonic frequency of the shifted peak. The spectrum thus consists of two superposed spectra of acoustic waves which have been formed at two different intervals of time during the development of the domain. The many peaks of acoustic power at rational multiples of the frequency of maximum linear net gain are formed at

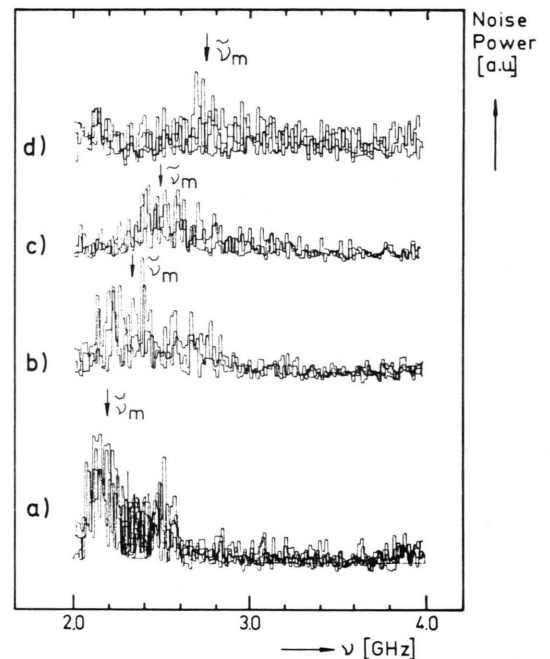


Fig. 2. Sequence of noise spectra showing the shift of the frequency of maximum gain. The parameter varied is the nonlinear current deviation ΔJ . The electronic attenuation used is also given.

- | | | |
|----|----------------------|-----------------------|
| a) | $\Delta J = 1.4$ A, | Attenuation = -20 dB, |
| b) | $\Delta J = 1.2$ A, | Attenuation = -20 dB, |
| c) | $\Delta J = 1.0$ A, | Attenuation = -20 dB, |
| d) | $\Delta J = 0.75$ A, | Attenuation = -10 dB. |

* During domain formation the activity of the medium rises rapidly from a situation $|\chi|/\epsilon_{TF} \ll 1$ to $|\chi|/\epsilon_{TF} \approx 1$ (see Sects. 4 and 5). In the latter regime, the Manley-Rowe relation is inappropriate. The authors owe this remark to H. G. REIK.

an early growth stage of the domain. At the further developed stage these waves propagate freely while the frequency of maximum gain is shifted due to the concentration nonlinearity. The main power is then concentrated in this frequency.

In Figs. 2 a–2 d successive shifts of the fundamental frequency peak are shown for several stages of the domain growth which is indicated by the magnitude ΔI of the deviation from the ohmic current. In Fig. 3 the frequency shift is plotted as a

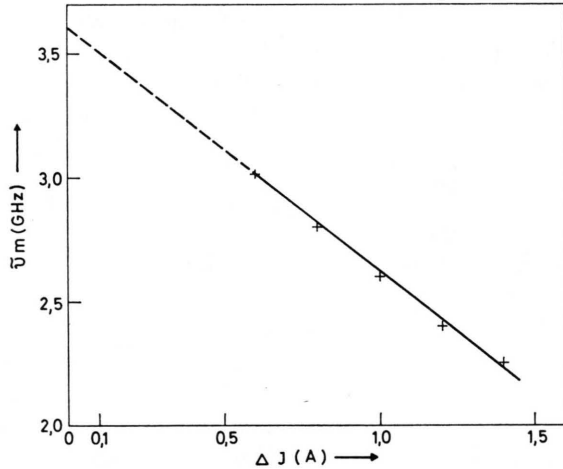


Fig. 3. Measured frequency of maximum nonlinear gain $\tilde{\omega}_m$ as a function of the nonlinear current deviation ΔI .

function of the nonlinear current deviation ΔI . The frequency shift is almost linear. An extrapolation to the linear situation $\Delta I = 0$ leads to the frequency 3.6 GHz for maximum linear net gain. The shifted frequencies are assumed to be frequencies of maximum nonlinear gain. This quantitative measurement of the frequency shift can then be inserted into a nonlinear theory to obtain a relation for domain parameters.

4. Theory

The subject of the frequencies of maximum nonlinear gain has been dealt with in various theories of nonlinear ultrasonic wave amplification^{10, 12–15}.

From the practical point of view, one would like to have a complete numerical account of the values of $\tilde{\omega}_m$. Such an account has been given by BUTCHER

and OGG¹² for their theory. For the theory in Ref. 10, this will be done here.

The following analysis utilizes the dielectric formulation of Ref. 10. It was shown that the nonlinear response of a collision-dominated solid state plasma is given by the dielectric function

$$\epsilon_{nl} = 1 + (\epsilon_{TF} - 1) \frac{1}{iy} \frac{J_1 - i\chi(2iy)}{J_1 - i\chi(2iy)}. \quad (1)$$

Here, $2y$ is the normalized amplitude of the potential variation

$$\delta\varphi = y k T \exp\{i Q x - i \omega t\} + c. c.$$

ϵ_{TF} is the Thomas-Fermi dielectric constant and the J 's are Bessel-functions. The activity parameter χ is given by $\chi = (1 - v_d Q/\omega) m^* \omega / (k T \tau Q^2)$ where m^* and τ are effective mass and conductivity relaxation time of the electrons, respectively. v_d is the electron drift velocity. The nonlinear temporal gain $\hat{\alpha}$ of an ultrasonic wave then is written

$$\hat{\alpha} = K^2 \omega \left[\frac{1}{i} \operatorname{Im} \frac{1}{\epsilon_{nl}} \right]. \quad (2)$$

K^2 is the electro-mechanical coupling-constant. Varying the frequency ($\omega = Q v_s$) and keeping the amplitude $2y$ fixed, the frequency $\tilde{\omega}_m$ of maximum nonlinear gain is found from the unique nontrivial root with respect to the variable $\epsilon_{TF} - 1 \equiv (\omega_m/\omega)^2$ of the equation

$$\operatorname{Im} \left\{ \frac{3 \epsilon_{nl} - 2 + \chi(\partial/\partial\chi) \epsilon_{nl}}{\epsilon_{nl}^2} \right\} = 0. \quad (3)$$

In Fig. 4, the computed values of $\tilde{\omega}_m/\omega_m$ are plotted as a function of the remaining parameters involved in ϵ_{nl} . As the activity parameter χ depends on $\tilde{\omega}_m$ the quantity χ_m is used instead of χ :

$$\chi_m = \left(\frac{\tilde{\omega}_m}{\omega_m} \right) \chi \equiv \left(1 - \frac{v_d}{v_s} \right) \frac{\omega_D}{\omega_m}. \quad (4)$$

$\omega_D = v_s^2/D$ is the diffusion frequency.

From Fig. 4, three regions a, b, and c may be distinguished. In region a, for small drift velocities or more precisely for $\chi_m^2 \ll 2y$, the values of $\tilde{\omega}_m/\omega_m$ are functions of $2y$ only. For $2y \gtrsim 10$, a simple power law $\tilde{\omega}_m/\omega_m = (y)^{-1/2}$ is valid. Such a relation has been proposed earlier in Ref. 12. This equation follows from Eq. (3) by the appropriate asymptotic expansion of the Bessel-functions in Eq. (2).

¹² P. N. BUTCHER and N. R. OGG, Brit. J. Phys. C 3, 706 [1970].

¹³ YU. V. GULAYEV, Sov. Phys. Sol. State 12, 328 [1970].

¹⁴ B. K. RIDLEY, Brit. J. Phys. C 3, 935 [1970].

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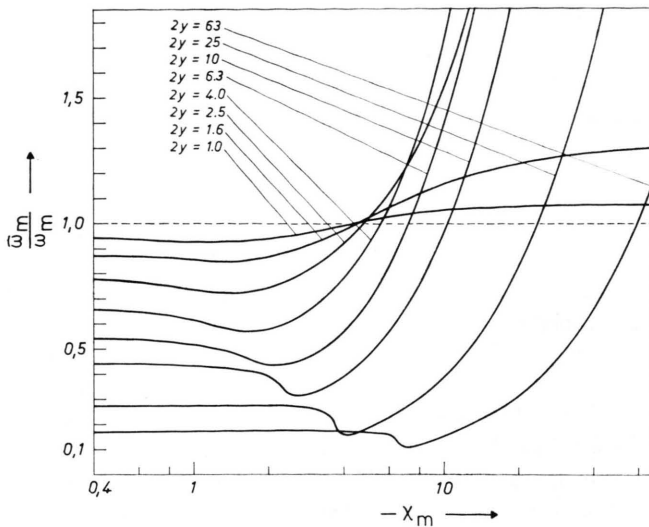


Fig. 4. Reduced frequency of maximum nonlinear gain as a function of the activity parameter $\chi_m = (1 - v_d/v_s) \omega_D/\omega_m$, and the potential ratio $2y$.

The transition region b is roughly given by $\chi_m^2 \lesssim 2y$. A simple analytic formula has not been found for it.

Region c is reached for $\chi_m^2 > 2y > 4$. In this region, the semi-logarithmic plot of Fig. 4 shows an exponential increase of $\tilde{\omega}_m/\omega_m$ with $\log |\chi_m|$. A more detailed analysis results in the simple relation

$$\tilde{\omega}_m/\omega_m = |\chi_m|/2y.$$

Because of Eq. (4) this is equivalent to

$$|\chi| = 2y.$$

This has been found earlier¹⁰ to be the location of gain maxima as a function of *intensity*. Here, it turns out, that this relation is valid for *frequencies* of maximum nonlinear gain. It is noted, that a relation $|\chi| \approx 2y$ is also valid for smaller potential ratios than those required for region c in a small strip extending slightly above the line $\chi_m^2 = 2y$.

5. Quantitative Evaluation of the Theory

For a direct comparison of the experimental results with the theory, the local dc electric field and the intensity of the shifted fundamental must be known. The local electric field can be deduced from the domain form, using the measured values of the

nonlinear deviation of the specimen current, but the equivalent potential ratio $2y$ is not available. The theory, therefore, is used to calculate this potential ratio by inserting the measured values of the frequency shift. The result of this procedure will be a relation between the local field, the shifted frequency and the bunching rate.

At first, the specimen data are compiled (n_0 electron concentration, T lattice temperature, μ electron mobility, v_s shear wave sound velocity, ω_m frequency of maximum linear gain, F_{synch} synchronous electric field, ω_D diffusion frequency, R^Q ohmic resistance, L sample length between contacts, U voltage across the sample, respectively):

Sample

GaAs G2/1322

$$\begin{aligned} n_0 &= 9 \cdot 10^{14} \text{ cm}^{-3}; & T &= 300^\circ \text{K}; \\ \mu &= 6700 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}; & F_{\text{synch}} &= 50 \text{ V cm}^{-1}; \\ v_s &= 3.35 \cdot 10^5 \text{ cm s}^{-1}; & \omega_m &= 2.4 \cdot 10^{10} \text{ s}^{-1}; \\ \omega_D &= 8 \cdot 10^8 \text{ s}^{-1}; & R^Q &= 390 \Omega; \\ L &= 1.08 \text{ cm}; & U &= 700 \text{ to } 750 \text{ V}. \end{aligned}$$

It is noted that in our case $(Ql) \lesssim 10^{-1}$, where Q is the sound wave number and l is the electron mean free path, respectively. Thus, hydrodynamic theories of sound amplification are appropriate and ω_m is calculated from the Hutson-White theory^{5,6}. The corresponding frequency is 3.8 GHz while the frequency of maximum linear net gain was deduced from the experiments to be 3.6 GHz. The 5 per cent downward shift is due to the nonelectronic loss. This loss is estimated to be $1 \cdot 10^6 \text{ s}^{-1}$ at 3.6 GHz in accord with the measured acoustoelectric current saturation and noise reduction measurements in Ref. 16. This attenuation value corresponds to a $\nu^{1.1}$ extrapolation of KELLER and ABELES's¹⁷ attenuation value 2 dB/cm at 0.63 GHz. These data are in accord with the above downward shift of the order of 5 per cent.

During the process of domain formation the gain is always large so that with decreasing frequency the influence of the nonelectronic loss on the downward shift should be less than 5 per cent. The nonelectronic loss will therefore be neglected in the following.

For the calculation of the local dc electric field a simple rectangular domain form of width ΔL is assumed. In this model, the correction $\Delta\chi$ inside

¹⁶ M. SCHULZ, to be published.

¹⁷ K. R. KELLER and B. ABELES, J. Appl. Phys. **37**, 1937 [1966].

the domain to the externally given value χ^{ext} is found to be

$$\Delta\chi = -\frac{\omega_D}{\omega} \frac{\mu}{v_s} R^\Omega \left(\frac{1}{\Delta L} - \frac{1}{L} \right) \Delta I.$$

Inserting the presented specimen data this formula reduces to

$$\Delta\chi = -\frac{4.2 \cdot 10^{10}}{\tilde{\omega}_m} \Delta I / A \text{ s}$$

for a domain width of 0.13 cm.

In Table 1, $\tilde{\omega}_m$, χ , χ_m , and $2y$ are compiled. $\tilde{\omega}_m$ is taken from Fig. 3. χ and χ_m are calculated by the above equation. Using the theoretical curves of Fig. 4 the potential ratio $2y$ is determined for various domain growth stages.

Table 1.

ΔJ [A]	$\tilde{\omega}_m$ [s ⁻¹]	$\frac{\tilde{\omega}_m}{\omega_m}$	χ	χ_m	$2y$	$\hat{\alpha}_{nl}$ [s ⁻¹]	$\hat{\alpha}_{lin}$ [s ⁻¹]
0	$2.27 \cdot 10^{10}$	0.95	-0.5	-0.5	0	$1.0 \cdot 10^7$	$1.0 \cdot 10^7$
0.6	$1.90 \cdot 10^{10}$	0.79	-1.3	-1.4	1.9	$1.6 \cdot 10^7$	$2.0 \cdot 10^7$
0.8	$1.76 \cdot 10^{10}$	0.73	-2.5	-1.8	2.5	$1.7 \cdot 10^7$	$2.2 \cdot 10^7$
1.0	$1.64 \cdot 10^{10}$	0.67	-3.2	-2.2	3.3	$1.6 \cdot 10^7$	$2.1 \cdot 10^7$
1.2	$1.52 \cdot 10^{10}$	0.62	-4.0	-2.6	4.0	$1.1 \cdot 10^7$	$2.0 \cdot 10^7$
1.4	$1.41 \cdot 10^{10}$	0.58	-5.0	-2.9	5.0	$0.9 \cdot 10^7$	$1.9 \cdot 10^7$

For information, the values of the nonlinear electronic gain computed by the theory in Ref. ¹⁰ and the linear gain in the μE_d -model ¹⁸ are given in the last two columns. The large nonlinear gains allow the noise distribution to adjust itself to the variable frequencies $\tilde{\omega}_m$ of maximum gain.

From the results of Table 1 it is seen, that a relation

$$|\chi| \approx 2y$$

¹⁸ A. MANY and I. BALBERG, Phys. Letters **24 A**, 705 [1967].

is always valid for the shifted fundamental frequency. The above result is only weakly dependent on the domain width ΔL within reasonable margins. The absolute values of χ and $2y$, however, strongly depend on ΔL .

6. Conclusions

From the presented results, the following model for the behaviour of acoustic frequencies involved in an acoustoelectric domain is suggested. During the very onset of domain formation the concentration nonlinearity weakly couples the acoustic wave at frequency of maximum linear gain to a wide variety of rational multiples of the fundamental frequency. Energy is pumped into subharmonic acoustic waves and mode-mixing with the fundamental frequency transfers energy into rational multiple frequencies.

In the strongly nonlinear regime the fundamental acoustic waves themselves are affected by the concentration nonlinearity. This results in a decoupling of the parametrically amplified signals and leads to a continuous downward shift in frequency of the amplified noise. The local domain field, the shifted frequency and the acoustic power at this frequency are related by $|\chi| \approx 2y$. This relation seems to be generally valid. It has also been found valid for a single frequency domain ¹¹, where the power is only related to the electric field because in that case the frequency is fixed.

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